

Atomic Energy Education Society- Distance Learning Programme

Class IX – Mathematics

Chapter 11 – Constructions

Hand Out Module 1/2

Construction is an important topic. It tells us how different shapes are made. It also teaches their implication and their academic relevancy. This chapter is divided in two modules. In the first module the introduction and some basic constructions will be discussed and in the second module some construction of triangle.

Introduction:

In earlier chapters we have drawn many diagrams to get a feeling of situation and as an aid for proper reasoning. They are called reference figures.

If one needs to draw an accurate figure, for example - to draw a map of a building , to design tools, and various parts of a machine, to draw road maps etc., some basic geometrical instruments are needed.

You must be having a geometry box which contains the following:

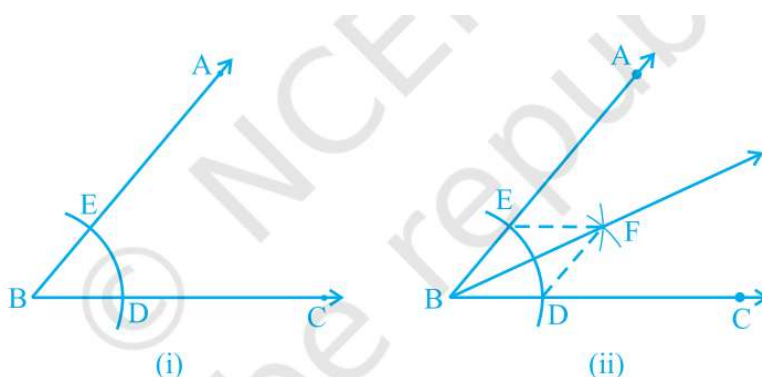
- (i) A graduated scale, on one side of which centimetres and millimetres are marked off and on the other side inches and their parts are marked off.
- (ii) A pair of set - squares, one with angles 90° , 60° and 30° and other with angles 90° , 45° and 45° .
- (iii) A pair of dividers (or a divider) with adjustments.
- (iv) A pair of compasses (or a compass) with provision of fitting a pencil at one end.
- (v) A protractor

A geometrical construction is the process of drawing a geometrical figure using only two instruments – an ungraduated ruler, also called a straight edge and a compass. In construction where measurements are also required, you may use a graduated scale and protractor also.

In this chapter, some basic constructions will be considered. These will then be used to construct certain kinds of triangles.

Basic Constructions:

Construction 1 : To construct the bisector of a given angle.



Given an angle ABC, we want to construct its bisector.

Steps of Construction :

1. Taking B as centre and any radius, draw an arc to intersect the rays BA and BC, say at E and D respectively [see Fig.].
2. Next, taking D and E as centres and with the radius more than $\frac{1}{2} DE$, draw arcs to intersect each other, say at F.
3. Draw the ray BF [see Fig.]. This ray BF is the required bisector of the angle ABC.

Justification:

Join DF and EF. In triangles BEF and BDF,

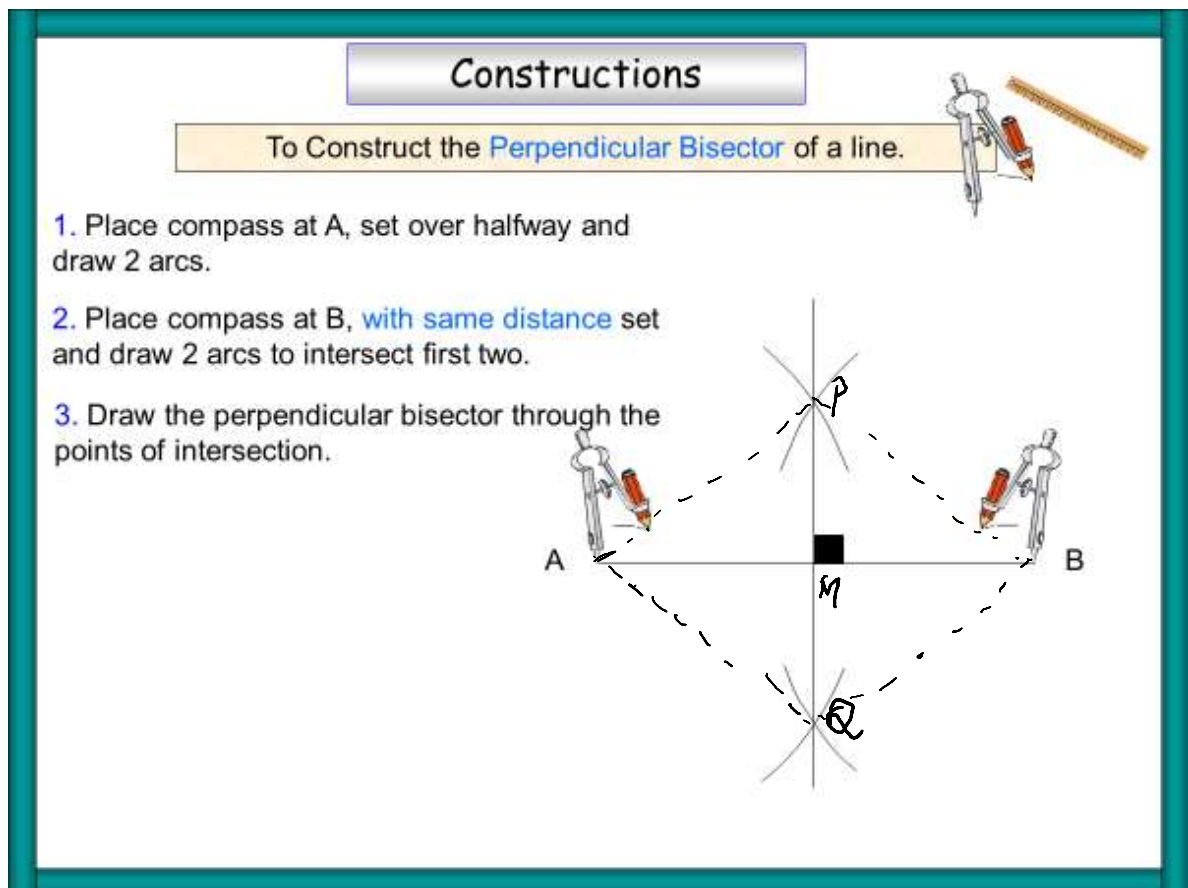
$BE = BD$ (Radii of the same arc)

$EF = DF$ (Arcs of equal radii)

$BF = BF$ (Common)

Therefore, $\triangle BEF \cong \triangle BDF$ (SSS rule)

This gives $\angle EBF = \angle DBF$ (CPCT)



To construct the perpendicular bisector of a given line segment.

Given a line segment AB, we want to construct its perpendicular bisector.

Steps of Construction :

1. Taking A and B as centres and radius more than $\frac{1}{2}$ AB, draw arcs on both sides of the line segment AB (to intersect each other).
2. Let these arcs intersect each other at P and Q. Join PQ (see Fig.).
3. Let PQ intersect AB at the point M. Then line PMQ is the required perpendicular bisector of AB.

Justification:

Join A and B to both P and Q to form AP, AQ, BP and BQ.

In triangles PAQ and PBQ,

AP = BP (Arcs of equal radii)

AQ = BQ (Arcs of equal radii)

PQ = PQ (Common)

Therefore, $\Delta PAQ \cong \Delta PBQ$ (SSS rule)

So, $\angle APM = \angle BPM$ (CPCT)

Now in triangles PMA and PMB,

AP = BP (As before)

PM = PM (Common)

$\angle APM = \angle BPM$ (Proved above)

Therefore, $\Delta PMA \cong \Delta PMB$ (SAS rule)

So, AM = BM and $\angle PMA = \angle PMB$ (CPCT)

As $\angle PMA + \angle PMB = 180^\circ$ (Linear pair axiom),

we get

$\angle PMA = \angle PMB = 90^\circ$.

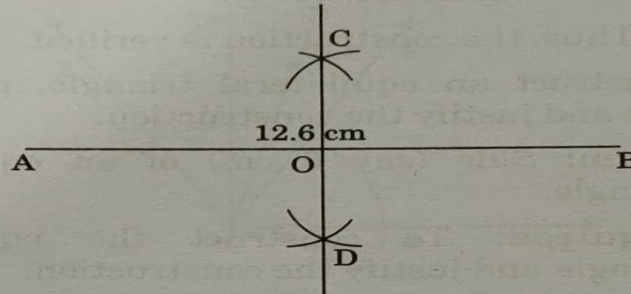
Therefore, PM, that is, PMQ is the perpendicular bisector of AB.

Let us solve some examples to understand the concepts.

✓ **Example 1:** Draw a line segment of length 12.6 cm, bisect it and measure each part.

Solution: Steps of construction:

- (i) Draw a line segment. $AB = 12.6$ cm.
- (ii) Taking A as centre draw an arc of radius more than $\frac{1}{2} AB$ on both sides of AB.

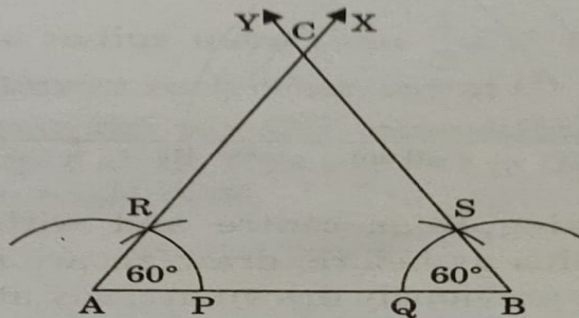


- (iii) Taking B as centre draw an arcs of same radius intersect previous arcs at C and D.
- (iv) Join CD, which bisects AB at O.
- (v) $AO = OB = 6.3$ cm.

✓ **Example 2:** Construct a triangle whose all angles are 60° each.

Solution: Steps of construction:

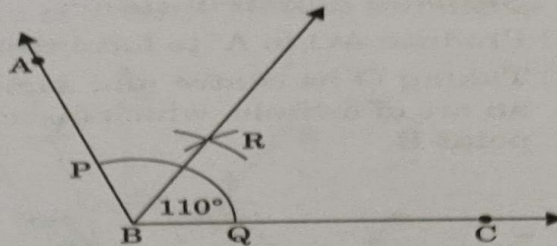
- (i) Take a line segment AB of any length.
- (ii) Taking A and B as a centre draw an arc of small radius which intersects AB at point P and Q.



- (iii) Taking P and Q as centres draw arcs of same radius, which intersects previous arcs at point R and S.
- (iv) Draw rays AX and BY passing through R and S respectively, which intersect each other at point C. Thus, ΔABC is a required triangle.

Example 3 Draw an angle of 110° with the help of protractor and bisect it. Measure each angle.
(NCERT Exemplar)

Solution:



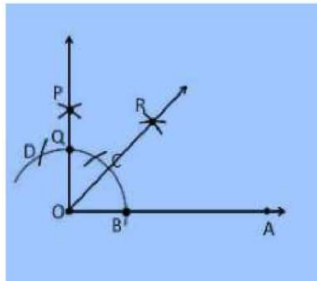
Steps of construction:

- (i) Draw an angle $\angle ABC = 110^\circ$ with the help of protractor.
- (ii) With B as centre and a convenient radius draw an arc to intersect the rays BA and BC at P and Q respectively.
- (iii) With centre P and a radius greater than half of PQ, draw an arc.
- (iv) With centre Q and the same radius as in (iii) draw another arc intersecting the previous one at R.
- (v) Draw ray BR.
Ray BR is the required bisector of $\angle ABC$.

Ex 4 2. Construct an angle of 45° at the initial point of a given ray and justify the construction.

Construction Procedure:

1. Draw a ray OA
2. Take O as a centre with any radius, draw an arc DCB is that cuts OA at B.
3. With B as a centre with the same radius, mark a point C on the arc DCB.
4. With C as a centre and the same radius, mark a point D on the arc DCB.
5. Take C and D as centre, draw two arcs which intersect each other with the same radius at P.
6. Finally, the ray OP is joined which makes an angle 90° with OP is formed.
7. Take B and Q as centre draw the perpendicular bisector which intersects at the point R
8. Draw a line that joins the point O and R
9. So, the angle formed $\angle ROA = 45^\circ$



Justification

From the construction,
 $\angle POA = 90^\circ$

From the perpendicular bisector from the point B and Q, which divides the $\angle POA$ into two halves. So it becomes
 $\angle ROA = \frac{1}{2} \angle POA$

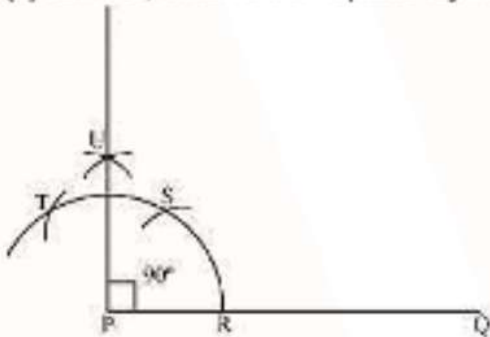
$\angle ROA = (\frac{1}{2}) \times 90^\circ = 45^\circ$

Hence, justified

Ex. 5 To construct an angle of 90 degree at the initial point of a given ray and Justify.

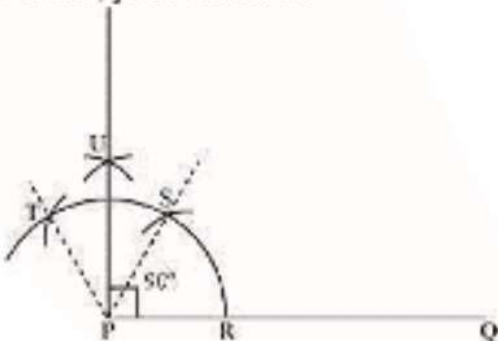
The below given steps will be followed to construct an angle of 90° .

- (i) Take the given ray PQ. Draw an arc of some radius taking point P as its centre, which intersects PQ at R.
- (ii) Taking R as centre and with the same radius as before, draw an arc intersecting the previously drawn arc at S.
- (iii) Taking S as centre and with the same radius as before, draw an arc intersecting the arc at T (see figure).
- (iv) Taking S and T as centre, draw an arc of same radius to intersect each other at U.
- (v) Join PU, which is the required ray making 90° with the given ray PQ.



Justification of Construction:

We can justify the construction, if we can prove $\angle UPQ = 90^\circ$.
For this, join PS and PT.



We have, $\angle SPQ = \angle TPS = 60^\circ$. In (iii) and (iv) steps of this construction, PU was drawn as the bisector of $\angle TPS$.

$$\therefore \angle UPS = \frac{1}{2} \angle TPS = \frac{1}{2} \times 60^\circ = 30^\circ$$

$$\begin{aligned} \text{Also, } \angle UPQ &= \angle SPQ + \angle UPS \\ &= 60^\circ + 30^\circ \\ &= 90^\circ \end{aligned}$$

Practice Time:

Q1. Construct the following angles and bisect them using ruler and compass

- a) 135° b) 37.5° c) 60°

Q2. Using scale and compass construct an equilateral triangle with one side as 4 cm.